Problems in laser physics

Sheet 7

Handed out on 14. 12. 17 for the Tutorial on 25. 1. 18

Problem 19: Q-switching (4P)

A Yb^{3+}:YAG laser emitting at $1029\ \mathrm{nm}$ uses a $250\ \mathrm{mm}$-long cavity. It has a 85%-reflectivity OC mirror and the internal losses were determined to $\Lambda = 2\%$.

(a) Show that a minimum pump power of three times the threshold pump power P_{th} is necessary in order to obtain at least an extraction efficiency of $\eta_e \approx 94\%$. (1P)
(b) Calculate the approximate Q-switch pulse width for the operation point in a). (1P)
(c) Calculate the final inversion $\langle \Delta N \rangle_f$ after the pulse ($\sigma_e(\lambda_s) = 2.31 \times 10^{-20}\ \mathrm{cm}^2$, $\sigma_a(\lambda_s) = 1.56 \times 10^{-21}\ \mathrm{cm}^2$, $\langle N \rangle = 1.38 \times 10^{26}\ \mathrm{m}^{-3}$) and derive the corresponding population inversions of the upper and lower laser manifold. (2P)

Problem 20: Acousto-optic modulators (4P)

In a Nd^{3+}:YAG laser ($\lambda_s = 1064\ \mathrm{nm}$) a quartz glass acousto-optic modulator is used for Q-switching. This material shows a refractive index of $n = 1.45$, a velocity of sound of $v_a = 3760\ \mathrm{m/s}$ and is driven by a rf signal at $50\ \mathrm{MHz}$. The acousto-optic figure of merit is $M_2 = 0.47 \times 10^{-15}\ \frac{\mathrm{s}^3}{\mathrm{kg}}$.

(a) Calculate the wavelength of the sound wave and show that a $50\ \mathrm{mm}$-long modulator satisfies the Bragg condition (1P).
(b) Calculate the Bragg angle and the external diffraction angle. (1P).
(c) Calculate the theoretical diffraction efficiency for a $50\ \mathrm{mm}$-long modulator equipped with a $5\ \mathrm{mm}$-wide transducer element creating $30\ \mathrm{W}$ of acoustic power (2P).

Problem 21: Electro-optic modulators and cavity dumping (4P)

A common material for a Pockels cell is KD_2PO_4 (KD^*P). It does not show a birefringence unless a longitudinal electric field E_z is applied. Then, the refractive indeces along the main axes of the index of refraction are given by

$$n_{x'} = n_0 + \frac{1}{2} n_0^3 r_{63} E_z \tag{1}$$

$$n_{y'} = n_0 - \frac{1}{2} n_0^3 r_{63} E_z \,, \tag{2}$$

wherein n_0 is the ordinary index of refraction and $r_{63} = 26.4 \times 10^{-6} \frac{\mu\mathrm{m}}{\mathrm{V}}$ is the electro-optic coefficient.

(a) Show that by applying a longitudinal voltage U_z across the Pockels cell, an incident radiation linearly polarized at $45°$ with respect to the x'-axis will acquire a phase difference of $\delta = \frac{2\pi}{\lambda_s} n_0^3 r_{63} U_z$ between the two orthogonal field components along the x'- and y'-axis. (1P)
(b) Assuming an incident linearly polarized radiation oriented at $45°$ between the positive x'- and y'-axis, show that the quarter- and half-wave voltages are given by (1P)

$$U_{\frac{\lambda}{4}} = \frac{\lambda_s}{4 n_0^3 r_{63}} \tag{3}$$

$$U_{\frac{\lambda}{2}} = \frac{\lambda_s}{2 n_0^3 r_{63}} \,. \tag{4}$$

These voltages correspond to a circularly-polarized output and a linearly-polarized output rotated by $90°$ with respect to the input polarization orientation, respectively.
(c) Calculate the half-wave voltage for a Nd^{3+}:YAG laser ($\lambda_s = 1064$ nm, $n_0 \approx 1.5$) and draw a setup for cavity dumping of that laser. (2P)
Hint: YAG is an isotropic laser medium and thus shows the same emission cross section for all polarisations.